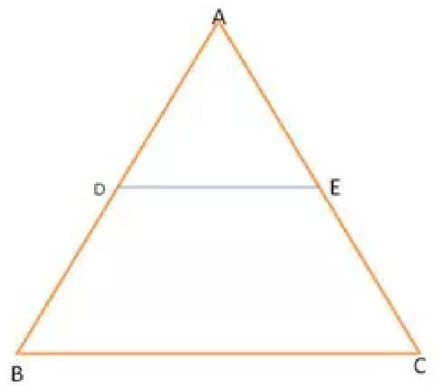
Chapter 15. Similarity

Ex 15.1

Answer 1.



Given :- $\frac{AD}{DB} = \frac{2}{7}$, AC=5.6

To find :- AE = x

Sol: In ΔABC, DE||BC,

∴ By BPT $\frac{AD}{DB} = \frac{AE}{EC}$

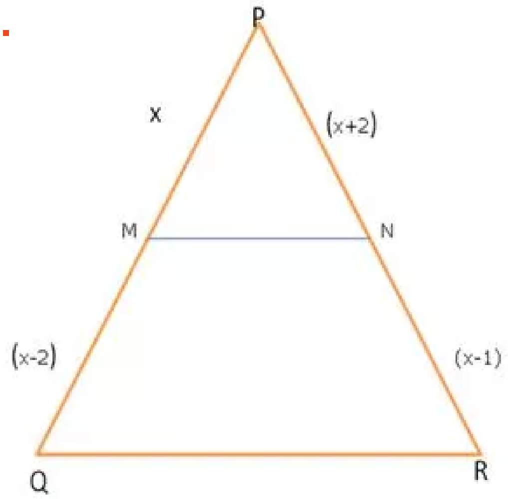
$$\frac{2}{7} = \frac{x}{5.6 - x}$$

$\Rightarrow$ 11.2-2x= 7x

$\Rightarrow$ 11.2 = 9x

$\Rightarrow$x = 1.24

Answer 2.



Sol: In ΔPQR, MN||QR,

∴By BPT $\frac{PM}{MQ} = \frac{PN}{NR}$

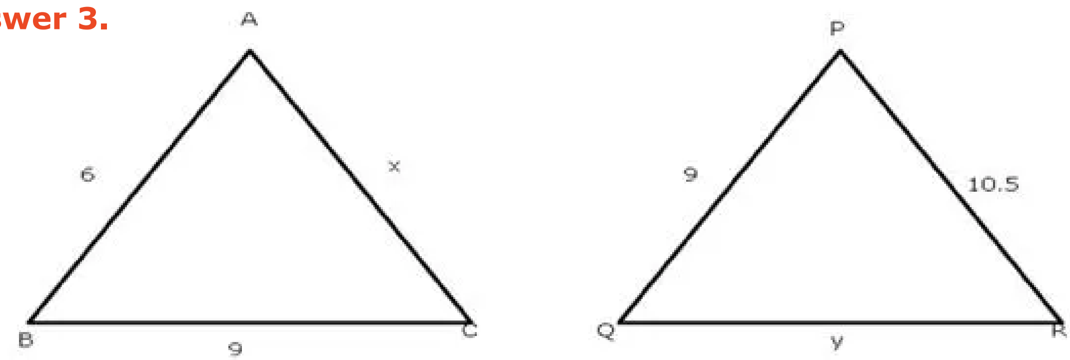
$$\frac{x}{x-2} = \frac{x+2}{x-2}$$

$\Rightarrow \quad x^2 - x = x^2 - 4$

$\Rightarrow$**-x = -4**

$\Rightarrow$x = 4

Answer 3.



Given:- $\Delta ABC \sim \Delta PQR$

To find:- AC and QR

Sol: $\Delta ABC \sim \Delta PQR$

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ (Similar sides of similar triangles)

$\frac{6}{9} = \frac{9}{y} = \frac{x}{10.5}$

$\frac{6}{9} = \frac{9}{y}$, $\frac{6}{9} = \frac{x}{10.5}$

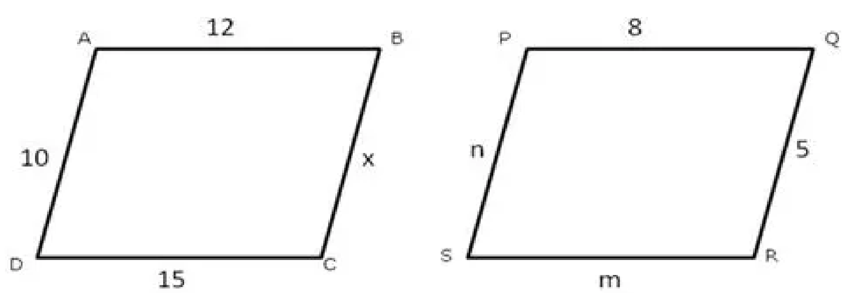
$\Rightarrow 6y = 81$ $\Rightarrow 63 = 9x$

$\Rightarrow y = \frac{81}{6}$ $\Rightarrow x = 7$

$\Rightarrow y = \frac{27}{2}$ $\Rightarrow AC = 7cm$

$\Rightarrow QR = 13.5cm$

Answer 4.



Given: quadrilateral ABCD~quadrilateral PQRS

To find: x, m and n

Sol: quadrilateral ABCD~quadrilateral PQRS

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{DC}{SR} = \frac{AD}{SR}$

$\frac{12}{8} = \frac{x}{5} = \frac{15}{m} = \frac{10}{n}$

$\frac{12}{8} = \frac{x}{5}, \frac{12}{8} = \frac{15}{m}, \frac{12}{8} = \frac{10}{n}$

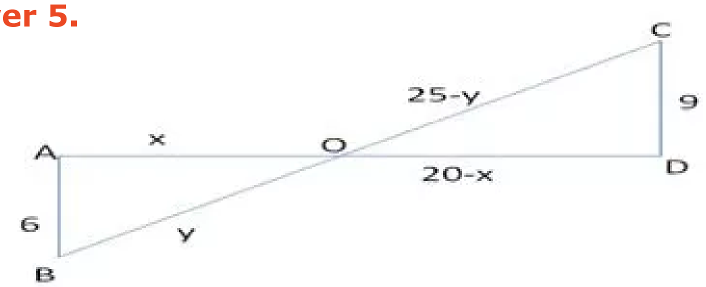
$60 = 8x$, $4m = 40$, $3n = 20$

$x = \frac{60}{8}$, $m = 10cm$, $n = \frac{20}{3}$

$x = \frac{15}{2}$, $m = 10cm$, $n = 6.66...$

$x = 7.5cm, m = 10cm, n = 6.67cm$

Answer 5.



To find: AO, BO, CO,DO

In ΔAOB and ΔCOD

$\angle OAB = \angle ODC$ (90°each)

$\angle AOB = \angle DOC$ (vertically opposite angles)

$\therefore \Delta AOB \sim \Delta DOC$ (AA corollary)

$$\therefore \frac{AO}{DO} = \frac{OB}{OC} = \frac{AB}{DC}$$

$$\frac{x}{20-x} = \frac{y}{25-y} = \frac{6}{9}$$

$$\frac{x}{20-x} = \frac{2}{3}, \frac{y}{25-y} = \frac{2}{3}$$

$3x = 40 - 2x,\ 3y = 50 - 2y$

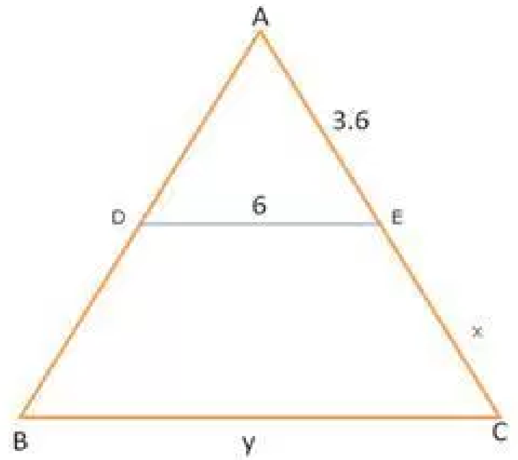
$5x = 40, 5y = 50$

$x = 8, y = 10$

AO = 8cm, OB = 10cm

OD = 20 - 8 = 12 cm , OC = 25-10 = 15cm

Answer 6.



Given: DE=6cm, AE=3.6cm, $\frac{AD}{DB} = \frac{2}{3}$, DE||BC

To find: BC and AC

Sol: In ΔABC, DE||BC

$\therefore$ By BPT $\frac{AD}{DB} = \frac{AE}{EC}$

$\frac{2}{3} = \frac{3.6}{x}$

$x = \frac{3.6 \times 3}{2}$

$= 1.8 \times 3$

$x = 5.4 = EC$

$\therefore AC = 3.6 + 5.4 = 9cm$

$AC = 9cm$

In ΔADE and ΔABC

$\angle ADE = \angle ABC$

Similarly $\angle AED = \angle ACB$ (corresponding angles)

$\therefore \Delta ADE \sim \Delta ABC$ (AA corollary)

$\frac{AE}{AC} = \frac{DE}{BC}$ (Similar sides of angles)

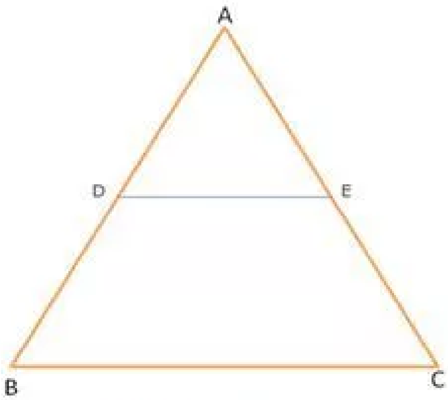
$\frac{3.6}{9} = \frac{6}{y}$

$y = \frac{9 \times 6}{3.6}$

$y = 15$

BC = 15cm

Answer 7.



To prove: DE||BC

Sol: AB=5.6cm AC=7.2cm

AD= 1.4cm AE=1.8cm

DB= 4.2cm EC= 5.4cm

$$\frac{AD}{DB} = \frac{1.4}{4.2} = \frac{1}{3} \quad ----(1)$$

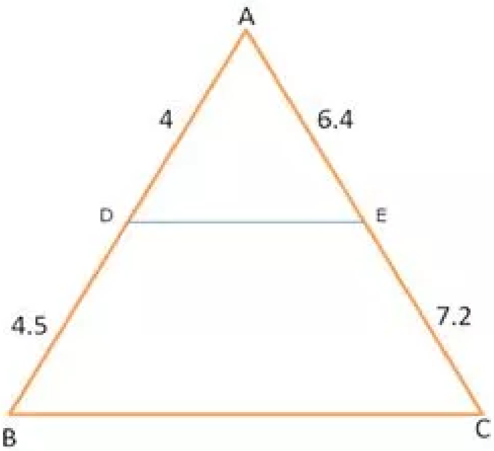
$$\frac{AE}{EC} = \frac{1.8}{5.4} = \frac{1}{3} \quad ----(2)$$

From (1) and (2)

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$\therefore$DE||BC (By converse of BPT)

Answer 8.



Sol: $$\frac{AD}{DB} = \frac{4}{4.5} = \frac{8}{9} \quad ----(1)$$

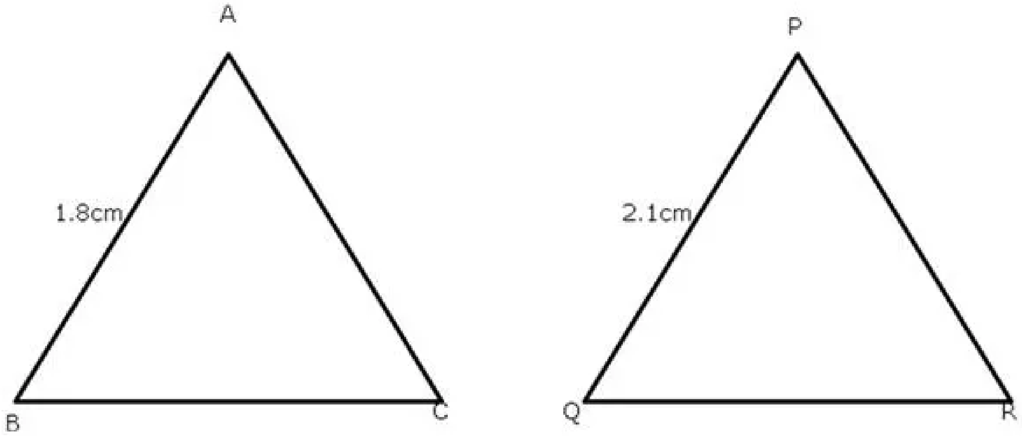
$$\frac{AE}{EC} = \frac{6.4}{7.2} = \frac{8}{9} \quad ----(2)$$

From (1) and (2)

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$\therefore$DE||BC (By converse of BPT)

Answer 9.



To find: $\frac{Ar\Delta ABC}{Ar\Delta PQR} = \frac{AB^2}{PQ^2}$ (The ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides)

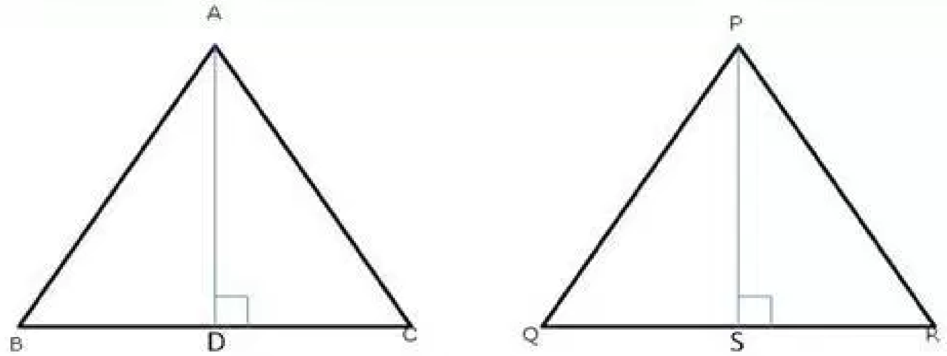
$$= \left(\frac{1.8}{2.1}\right)^2$$

$$= \left(\frac{6}{7}\right)^2$$

$$= \frac{36}{49}$$

Required ratio = 36 : 49

Answer 10.



Given: AD:PS=4:9 and $\Delta ABC \sim \Delta PQR$

To find: $\frac{Ar.\Delta ABC}{Ar.\Delta PQR}$

Sol: $\frac{Ar.\Delta ABC}{Ar.\Delta PQR} = \frac{AB^2}{PQ^2}$ ----(1)

(The ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides)

In ΔBAD and ΔQPS

$\angle B = \angle Q$ ($\Delta ABC \sim \Delta PQR$)

$\angle AOB = \angle PSQ$ ($90°$ each)

$\Delta BAD \sim \Delta QPS$ (AA corollary)

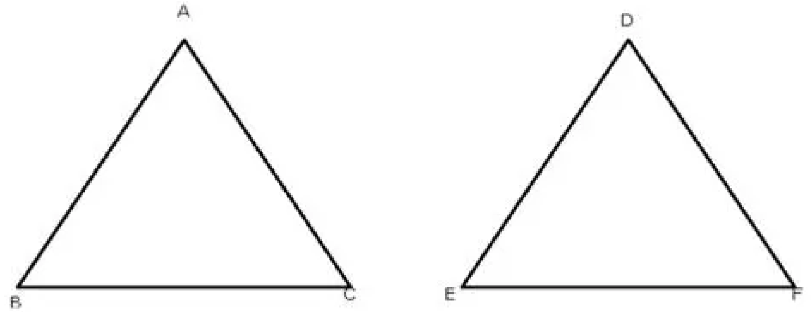
$\therefore \frac{AB}{PQ} = \frac{AD}{PS}$ ----(2) (Similar sides of similar triangles)

Using (1) and (2)

$$\frac{Ar.\Delta ABC}{Ar.\Delta PQR} = \frac{AD^2}{PS^2} = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

Required ratio is 16 : 81

Answer 11.



Given: $\Delta ABC \sim \Delta DEF$

To find: Ar. of ΔDEF

Sol: $\frac{Ar.\Delta ABC}{Ar.\Delta DEF} = \frac{BC^2}{EF^2}$ (The ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides)

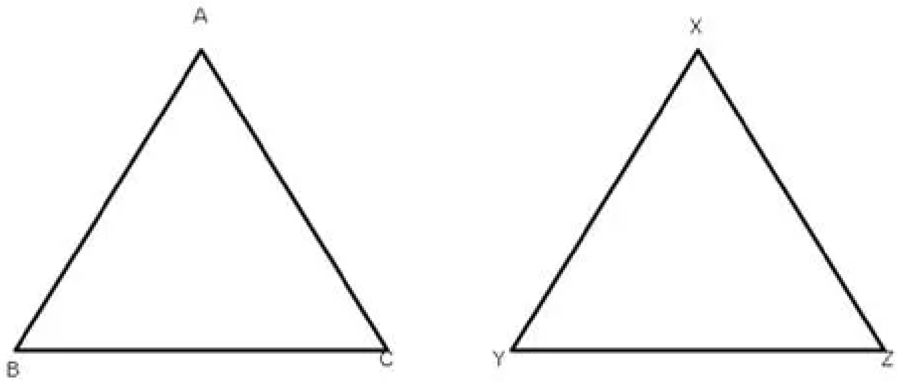
$$\frac{54}{Ar.\Delta DEF} = \left(\frac{3}{4}\right)^2$$

$$\frac{54}{Ar.\Delta DEF} = \frac{9}{16}$$

$$Ar.\Delta DEF = \frac{54 \times 16}{9}$$

$$= 96cm^2$$

Answer 12.



Given: $\Delta ABC \sim \Delta XYZ$

To find: YZ

Sol: $\frac{Ar.\Delta ABC}{Ar.\Delta XYZ} = \frac{BC^2}{YZ^2}$ (The ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides)

$$\frac{9}{16} = \frac{(2.1)^2}{YZ^2}$$

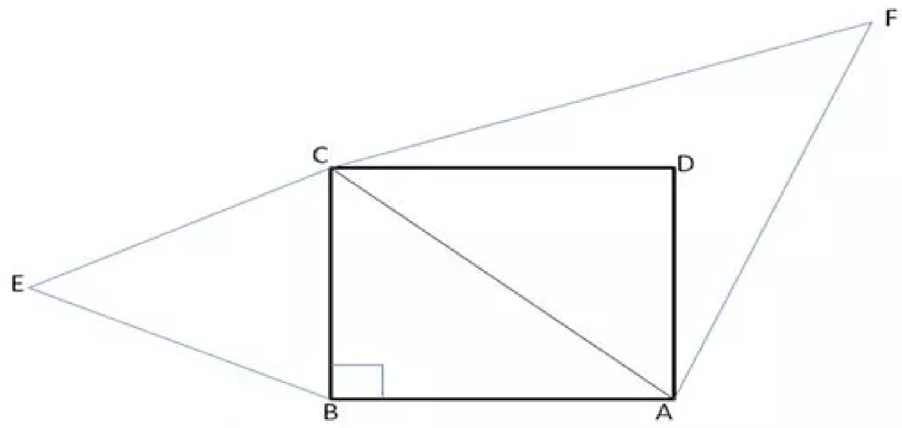
Taking square root both sides,

$$\frac{3}{4} = \frac{2.1}{YZ}$$

$$YZ = \frac{2.1 \times 4}{3}$$

$$YZ = 2.8cm$$

Answer 13.



In right triangle ABC,

By Pythagoras Theorem, $AB^2 + BC^2 = AC^2$

$2\ BC^2 = AC^2$ ---(1) ($\because AB=BC$)

Given, $\Delta BCE \sim \Delta ACF$

$$\frac{Ar.\Delta BCE}{Ar.\Delta ACF} = \frac{BC^2}{AC^2}$$ (The ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides)

$$= \frac{BC^2}{AC^2}$$

$$= \frac{1}{2}$$

Required ratio is 1 : 2

Answer 14.

(a) If AN : AC = 5 : 8, find ar(ΔAMN) : ar(ΔABC)

Given : $\frac{AN}{AC} = \frac{5}{8}$

To Find : $\frac{Ar.\Delta AMN}{Ar.\Delta ABC}$

In ΔAMN and ΔABC

$\angle AMN = \angle ACB$ (corresponding angles))

$\angle ABC = \angle ACB$

$\therefore \Delta AMN \sim \Delta ABC$ (AA corollary)

$$\therefore \frac{Ar.\Delta AMN}{Ar.\Delta ABC} = \frac{AN^2}{AC^2}$$ (The ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides.)

$$= \left(\frac{5}{8}\right)^2$$

$$\frac{Ar.\Delta AMN}{Ar.\Delta ABC} = \frac{25}{64}$$

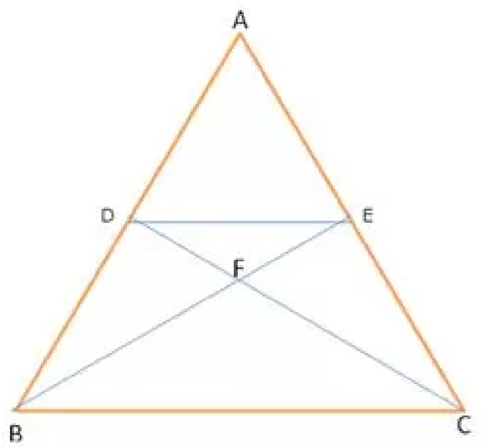
Required ratio is 25 : 64

(b) If $\frac{AB}{AM} = \frac{9}{4}$, find $\frac{Ar.(trapezium MBCN)}{Ar.(\Delta ABC)}$

ΔAMN ~ ΔABC {proved above}

$$\therefore \frac{Ar.\Delta AMN}{Ar.\Delta ABC} = \frac{AM^2}{AB^2} = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

Answer 15.



Given: $\frac{DE}{BC} = \frac{2}{7}$

To find: (Similar sides of similar triangles)

In ΔFDE and ΔFCB

$\angle FDE = \angle FCB$

$\angle FED = \angle FBC$ (Alternate interior angles)

$\Delta FDE \sim \Delta FCB$ (AA corollary)

$\therefore \frac{Ar.\Delta FDE}{Ar.\Delta FBC} = \frac{DE^2}{BC^2} = \left(\frac{2}{7}\right)^2 = \frac{4}{49}$ (The ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides)

Answer 16.

Given: $\frac{PT}{TR} = \frac{5}{3}$,

To find : $\frac{Ar.(\Delta MTS)}{Ar.(\Delta MQR)}$

Sol: In ΔPST and ΔPRQ

$\angle PST = \angle PQR$

$\angle PTS = \angle PRQ$ (Corresponding angles)

$\therefore \Delta PST \sim \Delta PQR$ (AA corollary)

$\therefore \frac{PT}{PR} = \frac{ST}{QR} = \frac{5}{8}$ (Similar sides of similar triangles)

Now, In ΔMTS and ΔMQR

$\angle MTS = \angle MQR$ (Alternate interior angles)

$\angle MST = \angle MRQ$

$\therefore \Delta MTS \sim \Delta MQR$ (AA corollary)

$\therefore \frac{Ar.(\Delta MTS)}{Ar.(\Delta MQR)} = \frac{TS^2}{QR^2} = \left(\frac{5}{8}\right)^2 = \frac{25}{64}$

i.e. 25 : 64 (The ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides)

Answer 17.

Given: $\frac{KL}{KT} = \frac{9}{5}$

To find: $\frac{Ar.\Delta KLM}{Ar.\Delta KTP}$

Sol: In ΔKLM and ΔKTP

$\angle KLM = \angle KTP$ (Given)

$\angle LKM = \angle TKP$ (Common)

$\Delta KLM \sim \Delta KTP$ (AA corollary)

$$\therefore \frac{Ar.\Delta KLM}{Ar.\Delta KTP} = \left(\frac{KL}{KT}\right)^2 = \left(\frac{9}{5}\right)^2 = \frac{81}{25}$$

i.e., 81 : 25 (The ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides)

Answer 18.

In ΔDEF and ΔGHF,

$\angle DEF = \angle GHF$ (90° each)

$\angle DFE = \angle GFH$ (Common)

$\Delta DEF \sim \Delta GHF$ (AA corollary)

$$\therefore \frac{Ar.(\nabla DEF)}{Ar.(\nabla GHF)} = \frac{EF^2}{HF^2} \quad ----(1)$$

(The ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides)

In right ΔDEF, (By Pythagoras theorem)

$DE^2 + EF^2 = DF^2$

$EF^2 = 10^2 - 8^2$

$EF^2 = 36$

$EF = 6$

From (1),

$$\frac{Ar.(\nabla DEF)}{Ar.(\nabla GHF)} = \left(\frac{6}{4}\right)^2 = \frac{9}{4}$$

i.e., 9 : 4

Ex 15.2

Answer 1.

Scale= 1 : 500

1cm represents 500cm

$$\frac{500}{100} = 5m$$

1cm represents 5m

$$\text{Length of model} = \frac{50}{5} = 10cm$$

$$\text{Breadth of model} = \frac{40}{5} = 8cm$$

$$\text{Height of model} = \frac{70}{5} = 14cm$$

Answer 2.

20cm represents 400m

$$\text{1cm represesnts } \frac{400}{20} = 20cm$$

$$\text{Width of model} = \frac{100}{20} = 5cm$$

Length of model = 20cm

$$\text{Surface area of the deck of the model} = 5cm \times 20cm$$

$$= 100\,cm^2$$

Answer 3.

Scale:- 1 : 500
1cm represents 500cm

$$= \frac{500}{100} = 5m$$

1cm represents 5m

(i) Actual length of ship= $60 \times 5m$

$= 300m$

(ii) $1\,cm^2$ represents $5m \times 5m = 25m^2$

Deck area of the ship $= 1500000m^2$

Deck area of the model $= \frac{1500000}{25}cm^2 = 60000cm^2$

(iii) $1\,cm^3$ represents $5m \times 5m \times 5m = 125\,m^3$

Volume of the model $= 200\,cm^3$

Volume of the ship $= 200 \times 125\,m^3$

$= 25000\,m^3$

Answer 4.

15cm represents = 30m

1cm represents $\frac{30}{15} = 2m$

$1\,cm^2$ represents $2m \times 2m = 4\,m^2$

Surface area of the model $= 150\,cm^2$

Actual surface area of aeroplane $= 150 \times 2 \times 2\,m^2$

$= 600\,m^2$

$50\,m^2$ is left out for windows
Area to be painted = 600 – 50
$=50\,m^2$
Cost of painting per m^2 = Rs. 120
Cost of painting $550\,m^2$ = 120×550
= Rs. 66000

Answer 5.

1cm on map represents 12500m on land

1 cm represents 12.5km on land

Length of river on map = 54cm

Actual length of the river = 54×12.5

$= 675.000\text{km}$

$= 675\text{km}$

Answer 6.

(i) Scale:- 1 : 200000

$\therefore$ 1cm represents 200000cm

$$= \frac{200000}{1000 \times 100} = 2\text{km}$$

1cm represents 2km

(ii) 1cm represents 2 km

$112^2 + 16^2$ represents $2 \times 2 = 4\text{km}^2$

(iii) 4km^2 is represented by km^2

1km^2 is represented by $\frac{1}{4}\text{cm}^2$

20km^2 is represented by $\frac{1}{4} \times 20\text{cm}^2 = 5\text{cm}^2$

Area on map that represents the plot of land= 5cm^2

Answer 7.

Actual area = $1872 km^2$

Area on map represents $117 cm^2$

Let 1cm represents x km

$\therefore 1 cm^2$ represents $x \times x$ km^2

Actual area = $x \times x \times 117$ km^2

$1872 = x^2 \times 117$

$$x^2 = \frac{1872}{117}$$

$x^2 = 16$

$x = 4$

$\therefore$ 1cm represents 4 km

Length of coastline on map = 44cm

Actual length of coastline = 44×4 km

= 176 km

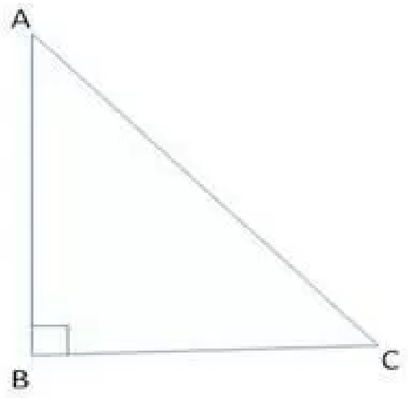
Answer 8.

Scale:- 1 : 25000

$\therefore$ 1 cm represents 25000cm

$$= \frac{25000}{1000 \times 100} = 2.5km$$

$\therefore$ 1cm represents 0.25km



Actual length of AB $= 6 \times 0.25$

$= 1.50$ km

Area of $\Delta ABC = \frac{1}{2} \times BC \times AB$

$$= \frac{1}{2} \times 8 \times 6 = 24cm^2$$

1cm represents 0.25 km

1 cm^2 represents $0.25 \times 0.25 km^2$

The area of plot $= 0.25 \times 0.25 \times 24 km^2$

$= .0625 \times 24$

$= 1.5 km^2$

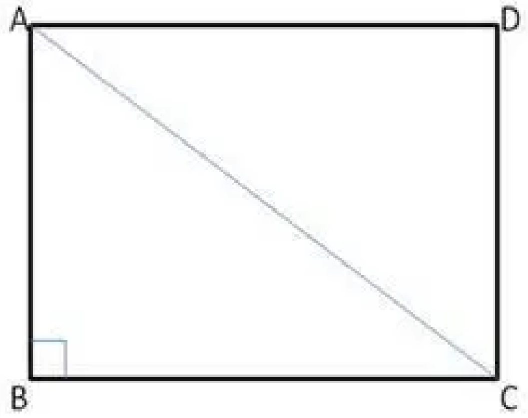
Answer 9.

Scale :- 1 : 25000

1 cm represents 25000cm

$$= \frac{25000}{1000 \times 100} = 0.25km$$

1 cm represents 0.25km



$AC^2 = AB^2 + BC^2$

$= 12^2 + 16^2$

$= 144 + 256$

$AC^2 = 400$

$AC = 20cm$

Actual length of diagonal = 20 × 0.25

= 5.00

= 5km

1 cm represents 0.25km

$1cm^2$ represents $0.25 \times 0.25 km^2$

The area of the rectangle ABCD = AB × BC

$= 16 \times 12 = 192 cm^2$

The area of the plot $= 0.25 \times 0.25 \times 192 km^2$

$= 12 km^2$